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An analytical and numerical study of failure waves

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Abstract

Based on recent observations in shock experiments on glasses, a new failure process has been suggested for a certain type of brittle solids, in which a failure wave propagates through a solid at some distance behind the compressive stress wave near but below the Hugoniot elastic limit. Since the failure wave phenomenon is different from the usual inelastic shock waves, a combined analytical and numerical effort is made in this paper to explore the impact failure mechanisms associated with the failure wave. Based on the experimental data available, it appears that the physical picture of failure wave is related to local dilatation due to shear-induced microcracking. A mathematical argument then leads to the conclusion that the failure wave should be described by a diffusion equation instead of a wave equation, which is in line with the bifurcation analysis for localization problems. However, the occurrence of different governing equations in a single computational domain imposes both an analytical and a numerical challenge on the design of an efficient solution scheme. With the use of a partitioned-modeling approach, a simple solution procedure is proposed for failure wave problems, which is verified by the comparison with data. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

Since Brar et al. (1991) and Kanel et al. (1991) reported the formation and propagation of a 'failure wave' in a series of impact experiments with glass plates and bars, continued efforts have been made to explore this interesting physical phenomenon (Bless and Brar, 1994; Bourne et al., 1995; Clifton, 1993; Grady, 1995a and b; Raiser and Clifton, 1994; Raiser et al., 1994; Rosenberg et al., 1996). No consensus can be made at the moment on the physics behind the failure wave phenomenon in amorphous materials, as indicated by a recent experimental and numerical study (Espinosa et al., 1997a and b). However, there exist enough data to show that some kind of progressive failure occurs in some glasses when they are shocked to near but below the Hugoniot elastic limit (HEL).

The essential feature of the failure wave phenomenon is that it propagates through a solid at

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some distance behind the compressive shock wave, and that the material behind and ahead of the failure wave is comminuted and intact, respectively. In fact, the peculiar character, that makes the failure wave seemingly different from usual inelastic shock waves (due to yielding or microcracking when the specimens are shocked beyond the HEL), is that the main quantities changed across the failure wave front are transverse stress for the increase in longitudinal motion, or transverse motion for the decrease in longitudinal stress (in non-uniaxial strain cases). It was also observed that the failure wave speed decreases with distance (Kanel et al., 1991). For geologic and other materials, the failure wave phenomenon has not been documented in the open literature.

Historically, work on shock waves has been performed with plate geometry (Zukas et al., 1992). The cases of plate impact generate a state of uniaxial strain but three-dimensional stress. The HEL is the maximum stress for elastic wave propagation under a uniaxial strain condition, without initiating material inelasticity. The shock response of glasses beyond the HEL often displays a two-wave like structure that has been interpreted as an inelastic behavior. Although the prompt microcracking across the compression wave front occurs if the glass specimen is shocked at or above the HEL, the macroscopic two-wave structure has suggested that the compressive failure might be suppressed by confining pressure at a shocked state beyond the HEL. The recent results about the delayed failure wave in the glass specimens, that are shocked to near but below the HEL, suggest that the HEL may not be an elastic limit, but rather, may be a transition in failure mechanisms. A possible transition is the one from a delayed kinetic-controlled failure process below the HEL to a prompt stress-controlled failure process above the HEL (Grady, 1995b). Another approach is to define the HEL as the stress level for which bulk glass undergoes permanent densification (Espinosa et al., 1997a). The formation and evolution of a failure wave may be the missing key in linking failure under shock compression to failure under tensile or unconfined compressive conditions. Other research results on dynamic failure of engineering materials further demonstrate that dynamic failure is an evolution process involving the transition among different modes, gradual softening with localized deformations, and possible phase transition, which might not be described simply by conventional strength theories (Bai and Dodd, 1993; Chen and Sulsky, 1995; Feng et al., 1998; Heuze, 1990; Rajendran, 1996; Rajendran et al., 1995).

Based on the experimental data available, it appears that the traditional question in dynamic failure analyses, 'What is the ultimate strength of solids?' should be reframed into 'What is the onset of a failure evolution process?' Identifying the onset of the dynamic failure and understanding the failure evolution process are what we will focus on in this paper. Specifically, the answers will be given to the following two questions:

(1) Is the failure wave just a manifestation of failure evolution in a particular class of materials under impact loading?
(2) Is the failure wave really a wave phenomenon?

Since the formation of localization is often the onset of material failure evolution, the previous research results on the formation and evolution of localization are briefly reviewed here to provide a solid foundation for the analysis procedure used later.

Generally, smeared and discrete approaches have been employed separately to predict the evolution of localization, although some attempts have been made recently to combine smeared and discrete models (Chen, 1996; Pradeilles-Duval and Claude, 1995). Among the smeared approaches proposed are nonlocal plasticity and damage models, rate-dependent models and Cosserat continuum models, all of which aim at describing the evolution of a continuously distributed microcracking process with the use of higher order terms in space and/or time, as reviewed by Chen and Schreyer (1994) with a focus on geologic materials. In the context of discrete approaches, a discontinuity in dependent variables is assumed in advance or introduced whenever a macrocrack occurs, and the subsequent crack propagation is traced based on the theory of fracture mechanics. No single existing approach appears to be able to predict the complete evolution process, in addition to some pressing limitations. In particular, the transition between smeared and discrete cracks is still not well-defined.

In fact, the key component of various modeling approaches is at attempt to predict the evolution of inhomogeneous interactions among material particles. In a macro-mechanical sense, however, the evolution process might be equally well characterized by the formation and propagation of a moving material surface associated with a local change in material properties due to microcracking. With the introduction of a moving material surface, a partitioned-modeling approach has been proposed for localization problems with one-dimensional analytical illustrations (Chen, 1993). The basic idea of the approach is that different local constitutive models are used inside and outside the localized deformation zone with a moving boundary being defined between different material domains. As a result, the extrapolation of material properties beyond the limitations of current experimental techniques can be avoided in identifying the evolution of localization, and simplified governing differential equations can be formulated in the partitioned domains for given boundary and initial conditions.

Recently, the approaches based on the jump conditions and on the transition between governing equations have been used separately to study plasticity and damage problems (Bebernes and Talaga, 1996; Brannon and Drugan, 1993; Pradeilles-Duval and Claude, 1995). However, the jump conditions and the transition between governing equations are in fact related to each other for localization problems. To establish a sound mathematical foundation for the partitioned-modeling approach, an attempt has been made to investigate the use of the jump forms of conservation laws in defining the moving material surface for rate-independent models, based on the transition between governing differential equations at a limit state (Chen and Sulsky, 1995). By taking the initial point of material failure as that point where the type of the governing differential equations changes, i.e., a hyperbolic to an elliptic type for dynamic problems and an elliptic to another elliptic type for static problems, a moving material surface can be defined through the jump forms of conservation laws across the surface. Jumps in density, velocity, strain and stress can be accommodated on this moving surface of discontinuity between two material domains. Interestingly, the problems involving the type change of the governing differential equations, accompanied by certain jumps in field variables, also occur in other areas such as fluid mechanics (Chen and Clark, 1995) and thermal shock wave propagation (Tzou, 1989, 1997). Because of its simplicity, the partitioned-modeling approach has found its way into several domain-transition problems associated with the jumps in field variables (Chen and Clark, 1995; Chen et al., 1997).

Although the approach based on the transition between governing equations accompanied by the jumps in certain field variables is meaningful from both physical and mathematical viewpoints, further research is required to investigate the applicability of the approach to general cases. Especially, a quantitative measurement of the moving surface of discontinuity must be available with a well-designed experimental procedure. However, this approach can be employed to answer qualitatively the above two questions related to a failure wave, as shown next.

2. Analyses

First of all, a commonly used procedure in impact dynamics (Zukas et al., 1992) is employed to formulate the jump forms of conservation laws in mass, momentum and energy, respectively. Then, the effects of jump conditions on the formation and evolution of a failure wave are investigated based on the physics involved, i.e., the local dilatation due to shear-induced microcracking. The effect of thermomechanical coupling is neglected here, which can be significant at strains exceeding 30%.

It should be pointed out that the procedure proposed here is different from that based on a propagation phase boundary (a transformation shock) (Clifton, 1993). As indicated by Espinosa et al. (1997a), no direct evidence had been given to support the hypotheses on the phase transformation within the glass, because of difficulties in recovering the shock samples. Currently, the authors are investigating the temperature change inside the glass specimens by using the thermochromic sensors. No conclusive results, however, have been obtained at this moment. In the paper, the jump in mass density is due to the shear-induced local dilation (microcracking), instead of the phase transformation.

Consider a plate of compressible material that has an initial state of internal energy E_0, pressure P_0, density ρ_0 and zero particle velocity. A uniform pressure P_1, that is suddenly applied to one face of the plate, would result in a wave traveling at velocity U_s if no failure occurs. The application of P_1 compresses the plate material to a new state of internal energy E_1, density ρ_1 and particle velocity $(U_p)_1$, as shown in Fig. 1 for a segment of the material normal to the direction of the shock front. Across the shock front, mass, momentum and energy must be conserved.

Conservation of mass across the shock front may be expressed by noting that after a short time period dt, the mass of material encompassed by the shock wave, $\rho_0 U_s A dt$, with A being cross-sectional area, now occupies the volume $\rho_1[U_s-(U_p)_1)Adt$ at density ρ_1, namely

$$\rho_0 U_s = \rho_1[U_s-(U_p)_1] \tag{1}$$

It follows from eqn (1) that the shock wave velocity is related to the density jump through

$$U_s = \frac{\rho_1 (U_p)_1}{\rho_1 - \rho_0} \tag{2}$$

Conservation of momentum can be found by noting that the rate of change of momentum of a mass of material $\rho_0 U_s A dt$ that is accelerated to $U_p = (U_p)_1$ from $U_p = 0$ in dt by a net force $(P_1 - P_0)A$ is given by

$$P_1 - P_0 = \rho_0 U_s (U_p)_1 \tag{3}$$

The expression for conservation of energy can be obtained by equating the work done by the shock wave with the sum of the increase of both kinetic and internal energy of the plate, i.e.,

$$P_1 (U_p)_1 = \frac{1}{2}\rho_0 U_s (U_p)_1^2 + \rho_0 U_s (E_1 - E_2) \tag{4}$$

Equations (1), (3) and (4) are the jump conditions that must be satisfied by the field variables on the two sides of a shock front. If the initial state is assumed to be known, eqns (2)–(4) contain a total of 5 unknowns (U_s, E_1, P_1, ρ_1 and $(U_p)_1$). Hence, the other three variables can be determined

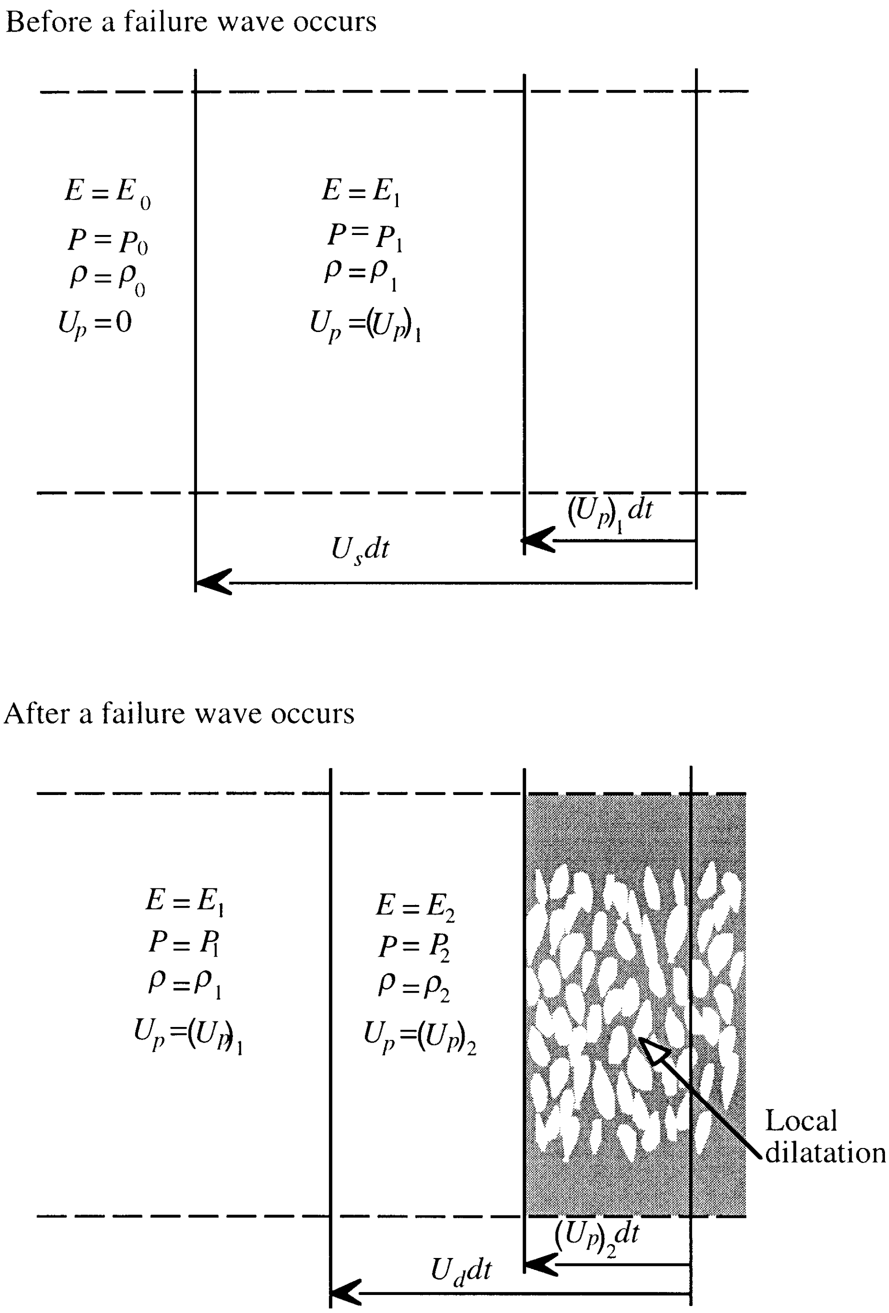

Fig. 1. The changes in field variables before and after a failure wave occurs.

if any two of the five variables can be measured or certain relationships between these variables are postulated. Elimination of the particle velocity from eqns (2) and (3) yields an expression for the shock velocity of the form

$$U_s^2 = \frac{\rho_1(P_1 - P_0)}{\rho_0(\rho_1 - \rho_0)} \tag{5}$$

As can be seen from eqn (5), U_s can be determined if the relationship between P_1 and ρ_1 is known for given pressure.

If a critical state is reached behind the shock wave, the failure evolution process could also be analyzed via the jump forms of conservation laws because every physical phenomenon must follow the conservation laws. To locate the failure front that follows the shock front, the same argument as that for the shock wave velocity U_s can be used based on the conservation of mass. The result is given by

$$\rho_1 U_d = \rho_2[U_d - (U_p)_2] \tag{6}$$

so that the velocity of failure evolution, U_d, is related to the density jump through

$$U_d = \frac{\rho_2(U_p)_2}{\rho_2 - \rho_1} \tag{7}$$

Similarly, the conservation of momentum takes the form of

$$P_2 - P_1 = \rho_1 U_d[(U_p)_2 - (U_p)_1] \tag{8}$$

and the expression for conservation of energy is given by

$$P_2(U_p)_2 = \frac{1}{2}\rho_1 U_d[(U_p)_2^2 - (U_p)_1^2] + \rho_1 U_d(E_2 - E_1) \tag{9}$$

Equations (7)–(9) contain a total of 4 unknowns (U_d, E_2, ρ_2 and P_2) if the State 1 is known and the continuity condition, $(U_p)_1 = (U_p)_2$, holds across the failure front.

As measured by Kanel et al. (1991) for the K8 glass specimens under plate impact loading, the longitudinal stress does not change significantly while the lateral stress approaches the longitudinal stress behind the failure wave front, which implies that the stress state behind the failure front approaches a hydrodynamic one. If there should be microcracks inside the compressed zone, local dilatation must occur as depicted in Fig. 1, which results in a jump in the longitudinal strain rate as observed by Rosenberg et al. (1996). In other words, the local dilatation (microcracking) must occur in a representative volume which is evolving due to the evolution of shear-induced localized microcracking under all negative principal stresses. Because of the local dilatation, the mass inside the representative volume would be pushed to the moving surface of discontinuity. As a result, the mass density in the representative volume near this moving surface would be higher than that on the other side of the moving surface. As shown, eqn (7), the moving surface can evolve with the speed U_d if $\rho_2 > \rho_1$, which causes the increase of lateral stresses behind the failure front. The physical picture based on the concept of local dilatation is in parallel with that on the free volume by Sundaram and Clifton (1996), with which the increase in free volume during the deformation process is related to the observed softening response. In fact, the idea on local dilatation is also in line with that proposed by Espinosa et al. (1997b) that microcracks are formed at a later time at the intersection of plastic flow surfaces inside the failure zone. Hence, the jump conditions can be used to investigate the formation and evolution of a failure wave, as shown next.

Because the continuity condition, $(U_p)_1 = (U_p)_2$, holds across the failure front, eqn (8) becomes

$$P_2 - P_1 = 0 \tag{10}$$

To examine how the jump in the longitudinal strain rate can be derivable from eqn (10), assume P_2 is related to P_1 by

$$P_2 = P_1 + T\Delta\varepsilon_2 \tag{11}$$

where T is the tangent stiffness for given strain increment $\Delta\varepsilon_2$ inside the failure zone. The use of eqns (10) and (11) then results in a special case of the classical necessary condition for a discontinuous bifurcation or loss of ellipticity (Chen, 1996). In other words, the continuity of the longitudinal stress across the failure front must result in the change in the types of governing equations, if there is a corresponding jump in the longitudinal strain rate.

The transition from a hyperbolic equation to an elliptic one is represented by a parabolic equation (John, 1982). Since the equation governing the dynamic response will change from being hyperbolic to being elliptic at a critical state if no higher order terms are introduced into the constitutive model (Chen and Sulsky, 1995), the failure wave front should be governed by a parabolic equation. Because a parabolic differential equation describes a diffusion process, the so called ‘failure waves’ should not be a wave phenomenon. As observed by Kanel et al. (1991), the speed of a failure wave decreases with distance, which implies that the evolution of a failure wave is diffusive. The proposed physical picture based on the concept of local dilatation is consistent with the mathematical argument about the transition between different equations, because local dilatation represents a transition point along the pressure-volume strain curve for pressure-dependent materials under triaxial compression conditions (Chen and Schreyer, 1990). It was observed from a recent experimental study (Espinosa et al., 1997a) that the longitudinal stress would progressively decay behind the failure wave front. This finding further supports the procedure proposed here, i.e., a hyperbolic equation would become an elliptic equation when failure occurs because of the sign change in tangent stiffness. Due to the sign change, the elliptic (softening) domain has a well-defined kinetics.

Although the failure wave phenomenon was observed only in shock experiments on glasses, it appears from the concept of local dilatation that other pressure-dependent materials, such as geologic materials, should also exhibit this kind of failure which might be more difficult to measure than glasses, due to the heterogeneity.

Because the jump conditions, eqns (7)–(9), involve 4 unknowns, an additional relationship must be established among these 4 variables. By treating the failure wave as a shear-induced microcracking (damage) diffusion process, the essential feature of failure waves under plate impact conditions can be predicted as demonstrated next.

3. Demonstrations

It is assumed that a failure wave originates from the sample surface after a critical state is reached, and that the shear-induced damage in a representative volume is governed by a strain-based damage surface,

$$f = \bar{e} - \bar{e}_0(1 + m_0 D) \tag{12}$$

where $\bar{e}$ is the second invariant of deviatoric strain tensor, $\bar{e}_0$ the critical state parameter, D damage and m_0 a model parameter. With the use of a standard procedure (Chen and Schreyer, 1990, 1994), it can be shown that the damage surface satisfies the thermodynamic restrictions and the rate of damage is determined by

$$\dot{D} = \frac{\dot{\bar{e}}}{\bar{e}_0 m_0} \tag{13}$$

Based on the experimental data available, it appears that the critical state is dependent on the thermomechanical energy dissipation. In other words, the critical state is reached if the energy dissipation in the representative volume reaches a critical value after a characteristic time, which results in the formation of a failure wave. If the functional relationship of the critical state is known, an eigen analysis of the acoustic tensor derived from the tangent stiffness tensor can be performed to identify the formation of the failure wave. However, there exist no data about the internal failure evolution with the energy dissipation. Hence, it is assumed here that a critical state is reached for a given impact load after a characteristic time. The evolution of damage is then given by eqn (13).

To represent the overall effect of local dilatation on the change in mass density, an integral average of damage over the representative volume, $\bar{D}$, is used to find ρ_2, namely,

$$\rho_2 = \rho_1 + (\rho_m - \rho_1)(1 - e^{-m_1 \bar{D}}) \tag{14}$$

with m_1 being a model parameter. As can be found from eqn (14), $\rho_1 = \rho_2$ if there is no damage, and ρ_2 will approach the maximum value ρ_m with the increase of $\bar{D}$. If ρ_m is reached, phase transformation might occur, a further discussion on which is beyond the scope of the paper.

The use of eqns (7) and (14) then results in

$$U_d = \left[\frac{\rho_1}{(\rho_m - \rho_1)(1 - e^{-m_1 \bar{D}})} + 1\right](U_p)_2 \tag{15}$$

As can be seen from eqn (15), U_d decreases with the increase of damage that diffuses from an initial defect D_0, and U_d finally approaches

$$\left[\frac{\rho_1}{\rho_m - \rho_1} + 1\right](U_p)_2$$

The recent research on the local limiting speed in dynamic fracture (Gao, 1996) might provide a clue to the limiting value of U_d.

If the rate equation governing the response inside the damage zone is taken to be elliptic, the rate of change in the response is simultaneous throughout the damage zone. Thus, three types of governing equations hold in the plate after a failure wave occurs: a hyperbolic equation between the shock front and damage front, a parabolic equation governing the damage front, and an elliptic equation inside the damaged zone. A simple constitutive model can be postulated for the relationships between the longitudinal stress and strain, and between the lateral stress and the longitudinal strain without invoking higher order terms, as follows:

$$\begin{cases} \sigma_{11} = \left[K_0(1+c_1D) + \dfrac{4}{3}G_0(1-c_2D) \right]\varepsilon_{11} \\ \sigma_{22} = \sigma_{33} = \left[K_0(1+c_1D) - \dfrac{2}{3}G_0(1-c_2D) \right]\varepsilon_{11} \end{cases} \tag{16}$$

where K_0 and G_0 are the initial bulk modulus and shear modulus, respectively, and c_1 and c_2 model parameters. Because there are no experimental data in the open literature, that show the evolving pattern of local dilatation associated with a moving surface of discontinuity, no attempt is made here to formulate a sophisticated constitutive model. Instead, simple formulations are employed to demonstrate how the essential feature of a failure wave can be qualitatively predicted with the use of the proposed procedure.

To overcome computational difficulties, several numerical schemes have been developed for impact problems (Camacho and Ortiz, 1996; Espinosa et al., 1996; Pijaudier-Cabot and Bode, 1995; Sulsky et al., 1994; Zhou et al., 1994). However, no existing schemes aim at the problems in which three types of governing differential equations occur in a single computational domain. Of course, this is not an easy task, because there exist many mathematical challenges such as the treatment of different boundary and initial data, and of different temporal and spatial scales.

Without attempting to hit three birds with one stone, a simple numerical procedure is proposed here to decouple the subdomains governed by different differential equations. In this procedure, an elastic wave equation (hyperbolic equation) holds and a Newmark time integrator is used in the whole domain, before a critical state is reached. After a failure wave originates from the sample surface, the subdomain ahead of the damage front is still governed by the elastic wave equation while the subdomain behind the damage front is governed by an elliptic equation due to eqn (16). The diffusing speed of the damage front is determined by eqn (15). A forward integration scheme is used in the elliptic (damaged) subdomain for given strain increments. In other words, the solutions in different subdomains are independent of each other, with the continuity of displacement holding across the damage front. As a result, conventional temporal and spatial discretization procedures for different differential equations can be used to obtain numerical solutions, as illustrated by sample problems.

To demonstrate the proposed solution procedure, the material parameters are assigned with the following values that are of the correct order of magnitude:

$E = 50$ GPa,	$v = 0.25$,	$\rho = 2500$ kg/m^3,	$D_0 = 0.01$
$c_1 = 1$,	$c_2 = 3.5$,	$m_0 = 1$,	$m_1 = 15$
$\bar{e}_0 = 0.04$,	$\rho_m = 5000$ kg/m^3		

A finite element mesh consisting of constant strain elements is used to discretize a plate of thickness 10 mm. A failure wave originates from the sample surface at $x = 10$ mm after a critical state is reached with the characteristic time being assumed to be 1 μs, as shown in Fig. 2 for the first element away from the sample surface. The jump in the longitudinal strain rate is shown in Fig. 3. As can be seen from Figs 2 and 3, the continuity of the longitudinal stress across the damage front is accompanied by a corresponding jump in the longitudinal strain rate, which results in the transition from a hyperbolic (wave) equation to an elliptic (softening or damaging) equation. The profiles of the effective stress and confining pressure are shown in Figs 4 and 5, respectively, for

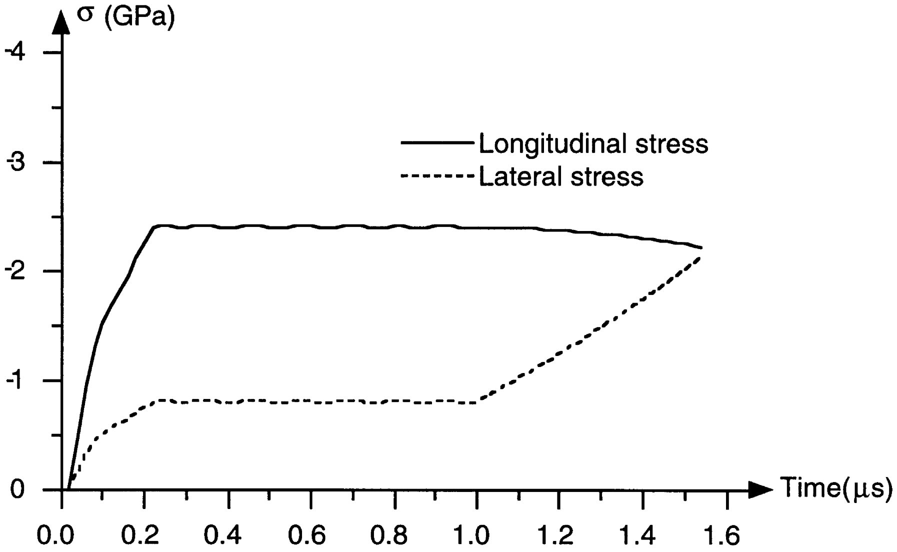

Fig. 2. The changes in longitudinal and lateral stresses after a critical state is reached.

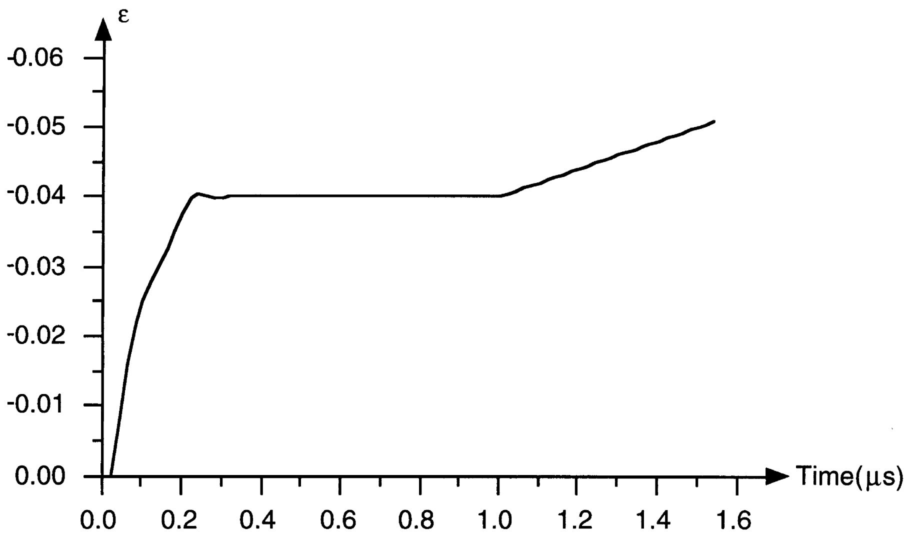

Fig. 3. The jump in the longitudinal strain rate corresponding to Fig. 2.

different time. These two figures demonstrate that behind the damage front the shear strength is lost and the stress state approaches a hydrodynamic one, which is qualitatively consistent with what was observed in shock experiments on glasses. A plot of D versus position at various time is given in Fig. 6, and the time history of damage near the impact surface is shown in Fig. 7. Figure 8 demonstrates that the damage front is diffusing from an initial speed, due to the initial defect, to a residual speed, due to the dissipation of energy.

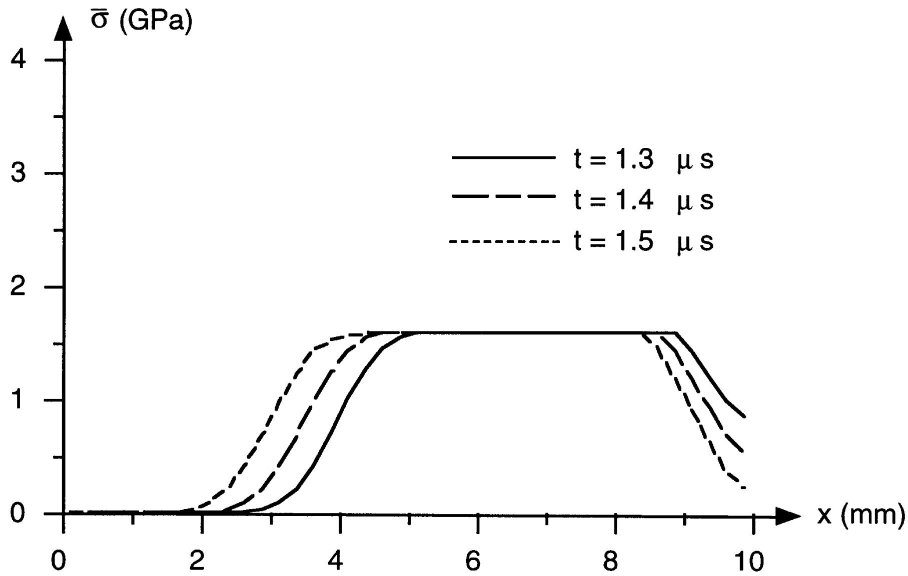

Fig. 4. The profiles of the effective stress at different time.

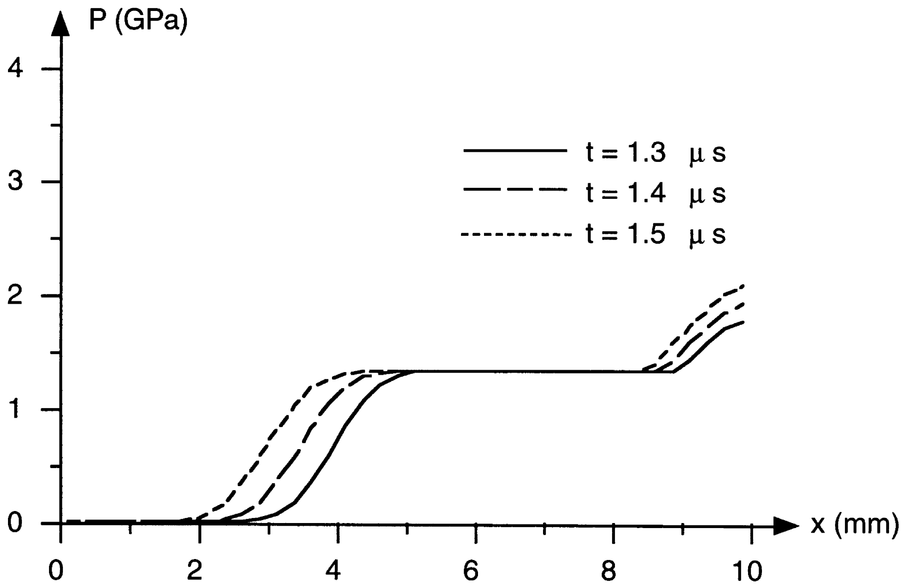

Fig. 5. The profiles of the confining pressure at different time.

Although impact experiments on geologic materials have not exhibited the failure wave phenomenon (He and Ahrens, 1994; Heuze, 1990; Sekine et al., 1995), the concept of local dilatation implies that the similar phenomenon must exist in geologic materials because their mechanical properties are usually pressure-dependent. Since the elastic limit is not well-defined for these

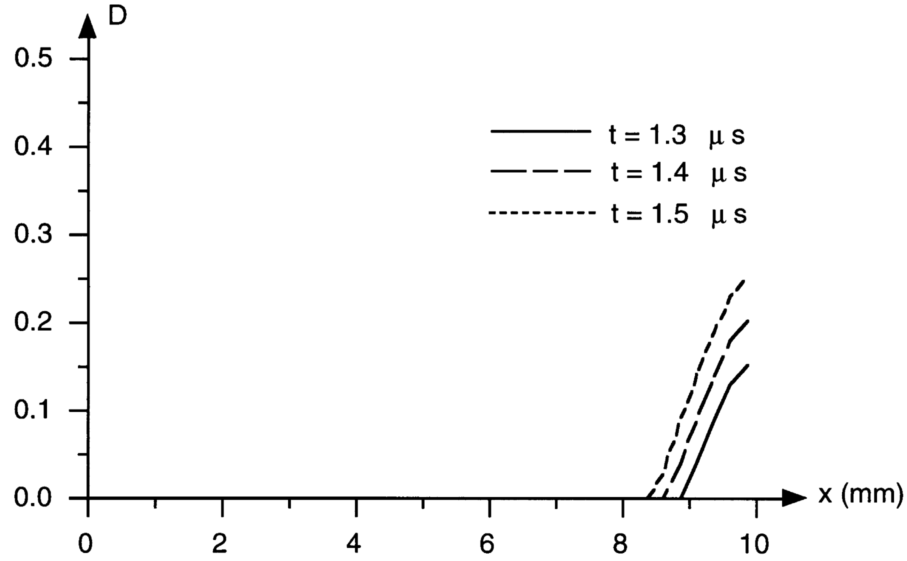

Fig. 6. The profiles of damage at different time.

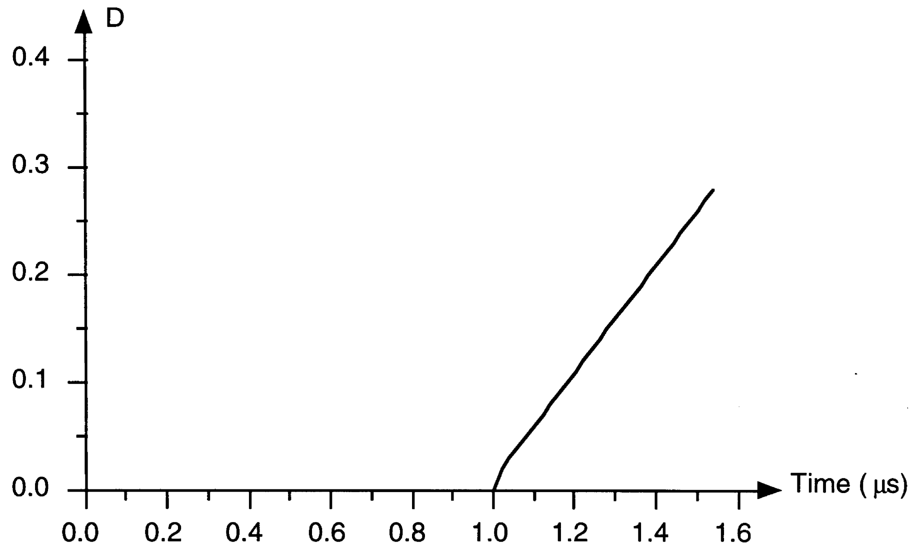

Fig. 7. The time history of damage near the impact surface.

materials, it might be more difficult to observe the transition from a delayed kinetic-controlled failure process below the HEL to a prompt stress-controlled failure process above the HEL, especially, the associated internal failure evolution.

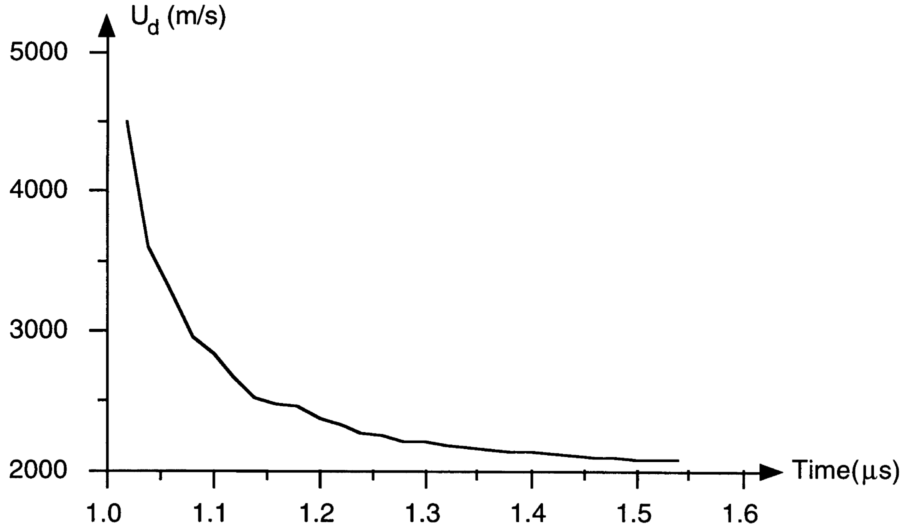

Fig. 8. The diffusing speed of the damage front.

4. Conclusions

According to the experimental data available, it appears that the physical picture of a failure wave is related to local dilatation due to shear-induced microcracking in an evolving representative volume. A mathematical argument, which is based on the transition between governing equations at a critical state that is accompanied by certain jumps in field variables, then leads to the conclusion that the evolution of a failure wave should be described by a diffusion equation instead of a wave equation. This conclusion is in line with the bifurcation analysis for localization problems. With the use of a partitioned-modeling approach, a simple solution procedure has been designed to predict the essential feature of failure waves under plate impact conditions.

However, the data obtained from well-designed experiments, that show the evolving pattern of local dilatation associated with a moving surface of discontinuity, are required to obtain the consensus on the physics behind the failure wave phenomenon, and to develop a detailed constitutive model with a parametric study. The occurrence of different governing equations in a single computational domain also imposes new challenges on the design of an efficient solution scheme for large-scale computer simulation. Future research is required to apply the proposed procedure to general cases.

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